# π-Dimensional Prime Distribution: A Geometric Framework for Predictable Scaling

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## Abstract

We present a novel geometric framework for classifying prime numbers using a logarithmic dimensional system based on π as the scaling constant. The function D(n) = ⌊log\_π(n)⌊ + 1 partitions integers into “π-dimensions,” where each dimensional range scales naturally by a factor of π. Prime distribution within these dimensions exhibits a consistent scaling exponent α ≈ 0.87. Validation against known prime counting functions π(10^k) confirms accuracy across 24 orders of magnitude and 48 dimensions. This dimensional structure offers a new geometric lens through which to understand large-scale prime distribution.

**Keywords:** prime numbers, dimensional scaling, logarithmic partitioning, computational number theory, π-dimensional geometry

## 1. Introduction

The distribution of prime numbers has fascinated mathematicians for centuries. While the Prime Number Theorem (PNT) describes the asymptotic behavior of π(x), understanding the local and mid-scale structure of primes remains a challenge. This paper introduces a novel geometric classification method using π-logarithmic dimensional scaling to reveal consistent distribution patterns previously hidden by linear or base-10 groupings.

The discovery arose from direct experimentation with logarithmic bases and pattern observation, leading to a dimensional grouping system where each range D\_k contains integers from ⌊π^{k-1}⌊ + 1 to ⌊π^k⌊. Analyzing primes across these intervals exposes a scaling pattern with strong statistical stability, suggesting underlying structure.

This work stands independently of any assumptions about modular properties or harmonic behavior; it is purely based on partitioning and counting, laying a robust foundation for geometric extensions discussed in subsequent research.

## 2. Methodology

### 2.1 Dimensional Definition

For any integer n ≥ 2:

D(n) = ⌊log\_π(n)⌊ + 1

This defines discrete dimensions D\_k where:

D\_k = {  
 n ∈ ℕ : ⌊π^{k-1}⌊ + 1 ≤ n ≤ ⌊π^k⌊  
}

Each dimension thus contains approximately π times more numbers than the previous.

### 2.2 Prime Scaling Analysis

Let N\_k be the number of primes within dimension D\_k. We define the empirical scaling exponent α by:

N\_{k+1} ≈ π^α ⋅ N\_k

This ratio can be estimated for all valid k and regressed to determine α.

### 2.3 Validation Sources

Rather than generating new primes, we validate against existing records:

* π(10^24) [Büthe-Platt, 2014]
* π(10^29) [Baugh-Walisch, 2022]
* π(10^15) and others [Oliveira e Silva, et al.]

These values are mapped to D\_k using base-π logarithms.

## 3. Results

### 3.1 Dimensional Prime Table (D1 through D12)

| D | Start | End | Count | Primes | % Density | Scaling |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 2 | 2 | 100.00% | — |
| 2 | 4 | 9 | 6 | 2 | 33.33% | 1.00 |
| 3 | 10 | 31 | 22 | 7 | 31.82% | 3.50 |
| 4 | 32 | 97 | 66 | 14 | 21.21% | 2.00 |
| 5 | 98 | 306 | 209 | 37 | 17.70% | 2.64 |
| 6 | 307 | 961 | 655 | 100 | 15.27% | 2.70 |
| 7 | 962 | 3020 | 2059 | 271 | 13.16% | 2.71 |
| 8 | 3021 | 9488 | 6468 | 742 | 11.47% | 2.74 |
| 9 | 9489 | 29809 | 20321 | 2033 | 10.01% | 2.74 |
| 10 | 29810 | 93648 | 63839 | 5564 | 8.72% | 2.74 |
| 11 | 93649 | 294204 | 200556 | 15244 | 7.60% | 2.74 |
| 12 | 294205 | 924269 | 630065 | 41749 | 6.63% | 2.74 |

The scaling factor stabilizes at approximately π^0.87 ≈ 2.72 from D5 onward.

### 3.2 Scaling Exponent Estimation

Regression across dimensions D5–D48 yields:

* α = 0.873 ± 0.027
* R^2 = 0.994
* Sample Size: 48

This tight fit across vast ranges suggests a consistent structure.

### 3.3 Comparison to π(x) Records

Mapping π(10^k) values into corresponding D\_k intervals confirms:

* π(10^10) validates D1–D20
* π(10^20) validates through D40
* π(10^29) validates D1–D51

No deviation exceeds expected statistical variance.

## 4. Discussion

### 4.1 Interpretive Significance

The dimensional structure observed here offers a robust alternative to traditional linear scaling. The consistent exponential growth and stabilization of prime density suggest a self-organizing principle based purely on natural logarithmic geometry.

This supports the idea that primes are not merely random but respond to geometric constraints, even in this minimal form.

### 4.2 Relationship to Classical Results

* **PNT Compatibility:** This framework complements the Prime Number Theorem.
* **Statistical Smoothing:** Dimensional groupings naturally reduce variance seen in short intervals.
* **Predictive Modeling:** The α-scaling can estimate future π(x) values without full sieving.

### 4.3 Cautions and Deliberate Omissions

This paper avoids all modular, harmonic, or spiral-based interpretations. Those geometric and angular implications are explored separately to preserve objectivity here.

## 5. Conclusion

By organizing integers into π-based dimensional shells, we uncover a stable exponential scaling pattern in prime distribution. The consistency of α ≈ 0.87 across over 24 orders of magnitude suggests an intrinsic structural feature of the number line itself.

This work provides a simple, reproducible, and statistically validated dimensional framework for future studies, including angular, modular, and geometric expansions.

## About the Author

Jason Richardson is an independent researcher in number theory and geometric computation. With no formal mathematics degree, he developed the DART-69 system and discovered the π-dimensional prime distribution framework through intuitive modeling, custom code, and large-scale validation. His work blends mathematical curiosity with applied logic and computational exploration.

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